Workshop:



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8 May 2016

Outline

- **Part I** Basic notions:
 - Modeling and simulation
 - Crash intro to Graph Theory
- **Part II** a **Boolean** model for regulatory network simulation
- Part III BioNSi tool hands-on

Mathematical Models and Computer Simulation

• Simulation: an imitation of how a real-world process or system operates over time



A computer simulation (aka *in silico* experiments) is a simulation run on computers.

In biology, computer simulation is used to replace / complement some tedious and costly lab experiments.
 It enables conducting numerous "experiments" under various conditions, in a scale that is infeasible experimentally.

3

Mathematical Models and Simulation (cont.)

- Running a computer simulation requires constructing a mathematical model some formal representation of the system using, e.g., equations and algorithms.
- A model is an abstraction of reality.

A valuable model should capture the relevant aspects of the system, with the appropriate level of detail.

- Advantages of using math. models and simulation in biology:
 - 1) Descriptive: forces clarity of expression and precision in describing systems / processes / hypotheses
 - 2) Analytic: promotes understanding and provides insights
 - 3) Predictive: enables predictions regarding the behavior of the system under various conditions

Quantitative vs. Qualitative Models

- Qualitative^{*} models predict trends, types of dynamics, and general properties, such as:
 - Robustness to mutations
 - Stability against perturbations (e.g., change in a cell's conditions)
 - Fail-safety
 - Conditions for cyclic behavior
- Quantitative^{**} models predict specific values that can be compared to actual experimental data, such as:
 - Kinetics
 - Concentrations
 - Expression levels

Continuous vs. Discrete Models

- **Continuous**^{*}: infinitely divisible (*e.g.* reals)
 - Usually in the form of differential equations
 - Provide a high resolution description of the biological system



- **Discrete**^{**}: made of distinct, indivisible units (*e.g.* integers)
 - Discrete time: time progresses in discrete steps (clock tics)
 - Discrete space: biological quantities are discrete
 - Discrete models tend to be simpler, more computationally efficiency, and require less detailed biological data.



Continuous vs. Discrete Models (cont.)

- What about biological quantities? Are they discrete or continuous?
 - *e.g.*: interactions, concentrations, reaction times, signals, etc.
- Whether these are really continuous or discrete is a physical question, or maybe even a philosophical one.
 But anyway, modeling does not have to conform with the nature of the modeled entity.

7

Networks



Biological Networks - Examples



Metabolic and amino acid biosynthesis pathways of yeast

Schryer et al., BMC Systems Biology, (2011)



E. coli transcriptional regulatory network

Guzmán-Vargas et al., BMC Systems Biology (2008)



The PPI Network in yeast Jeong et al., Nature (2001)



Klipp et al., BMC neuroscience (2006).

Visualization and Analysis of Biological Networks

- There are various tools and software packages for the visualization of networks.
- When the networks are large and dense, it is sometimes difficult to extract meaningful information from their visual representation.
- Computational analyses of networks enable valuable insights into their structure, properties and behavior.
- The mathematical structure used to model networks is called a graph. Graph theory deals with studying various aspects of graphs.

Introduction to Graph Theory

• A graph is a set of interactions, or relationships, between pairs of objects.



• The objects are called **nodes**^{*}, and the interactions are termed **edges**^{**}.

• If the edges have directions, the graph is called a directed graph (or digraph). Otherwise it is an undirected graph.

¹¹ * or vertices (sg. vertex). Hebrew: צומת / קדקד

** or arcs, links, chains. Hebrew: קשת / צלע

Graphs – More Formally

- A graph G is a pair G = (V, E) where:
 - V is a set of element (called nodes)
 - E is a set of pairs from V (called edges)



• In an undirected graph we ignore the order of nodes in an edge.

Basic Notions



 $V = \{a,b,c,d\}$

$$E = \{(a,b), (a,c), (c,c)\}$$



- Common notations: $|\mathbf{V}| = \mathbf{n}$, $|\mathbf{E}| = \mathbf{m}$
- <u>neighboring</u> / <u>adjacent</u> /<u>connected</u> nodes
- <u>neighborhood</u> of a node
- <u>degree</u> of a node (for directed graphs: in-degree and out-degree)
- a <u>loop</u>

Weighted Graphs

• A weighted graph is a graph in which edges are assigned values, called weights.



• What can weights resemble in a biological context?

Paths and Connectivity

 A path p in a graph G = (V, E) is a sequence of nodes p = (v₁, v₂, ..., v_k) Such that (v_i, v_{i+1}) ∈ E for every 1≤i<k



If $v_1 = v_k$ then the path is called a cycle.

• The length or weight of a path *p* is the number of edges in it. In weighted graphs, this is the sum of weights along the path.

• A graph is connected if there is a path from every node to every other node (in other words every node is reachable from any other node)

Special Graphs

• <u>Tree</u>

An undirected graph that is:

- connected
- acyclic (= contains no cycles)



• <u>Rooted tree</u>

A tree with a special node called root. This defines a hierarchy:

- parents and ancestors
- children and descendants

A leaf is a node with no children.





In CS, rooted trees grow downwards...

(A full binary tree with 16 leaves. Courtesy of Dr. Shlomit Pinter, photo taken in Kenya, 2005)

Graph Representation

• One simple way to represent a graph is a matrix of adjacencies (there are additional ways that we will not discuss).

$$G = \begin{bmatrix} 0, -2, 1, 0 \end{bmatrix}, \\ \begin{bmatrix} -2, 0, 0, 0 \end{bmatrix}, \\ \begin{bmatrix} 1, 0, -1, 0 \end{bmatrix}, \\ \begin{bmatrix} 0, 0, 0, 0 \end{bmatrix} \end{bmatrix}$$

• For example, G[1][0] == -2, and G[3][1] == 0.

The Bigger Picture

• Graph theory and graph algorithms are very central within CS.

Computational biologists use graph theory to study properties of biological networks, and graph algorithms to solve biological problems (some examples next).

Common Problems in Graph Theory

• The shortest path problem: find a path from *s* to *t*, whose "cost" is minimal.

The maximal flow problem: find a maximum feasible flow from *s* to *t*.
 (weights are flow capacities).

- The spanning tree problem: find a subgraph that is a tree and connects all the vertices, with minimal total weight.
- You may also want to check out these two famous topics, related to graph theory: the <u>7 bridges of Königsberg</u>, and the <u>4 color theorem</u>.







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Boolean Model for Regulatory Networks

- Boolean 0/1
- A simple case of a discrete model
- Qualitative

Based on the paper: <u>The yeast cell-cycle network is robustly designed</u>, Li et. al., PNAS 2004

The Boolean Model – User Input

- The model consists of a graph with states.
- <u>Nodes</u>: can represent proteins, mRNA, nutrients, cellular events (*e.g.* mitosis), external signals (*e.g.* light, injected hormone)
 - Can assume state 0 (non active) or 1 (active)
 - A vector of the network is a sequence of all nodes' states: [0,1,1,1]
 - Each node is given an initial state. So the network has an initial vector.
- <u>Edges</u>: regulation effects weighted (+ activation) (- inhibition)



The Boolean Model - Simulation

• <u>Time</u> is discrete (time steps = 1, 2, 3, ...)

- A <u>transition function</u> determines the states of nodes in the next time step in a synchronous fashion.
 It moves the system into the next vector.
- Transition function is applied repeatedly, until one of two options:
 - Steady state (aka "fixed point") 2 consecutive identical vectors

□ Infinite loop

2 non-consecutive identical vectors



The Boolean Model – Transition Function

• <u>Transition function</u>:



• The state update:

$$s_i(t+1) = \begin{cases} 1 & \text{if } \sigma_i(t) > 0\\ 0 & \text{if } \sigma_i(t) < 0\\ s_i(t) & \text{else} \end{cases}$$

Example 1

• Let's see what happens to node A at t₂:



	A	В	С	D
t ₁ :	1	1	0	0
	+		+	_
t ₂ :	0			

= -1

Example 2

• Let's see what happens to node A at t₂:





Example 3

• Let's see what happens to node A at t₂:





A Simulation - Full Example

	A	В	С	D	Е	F
t ₁ :	1	0	1	0	0	0
t ₂ :	0	0	1	1	0	0
t ₃ :	0	0	1	1	1	0
t ₄ :	0	0	1	1	1	1
t ₅ :	0	0	1	0	1	1
t ₆ :	0	0	1	0	1	1



Vectors in t_5 and t_6 are identical \rightarrow steady state.

Can the system get out of this steady-state in the future?

Exercise - Loops

Give an example for a network and initial vector that yield an infinite loop.

Hint: 2 nodes are enough.

Case Study: The Cell-Cycle in Yeast

"The yeast cell-cycle network is robustly designed", Li et. al., PNAS 2004

- 11 nodes main regulators of yeast cell-cycle.
- "Cell Size" is the signal for entry into cell-cycle
- Each node can be either 0/1.
 Red/yellow edges: weight = -1
 Green edges: weight = +1



• Simulation is executed on all possible initial vectors. How many?

How many potential fixed points?

Yeast Cell-Cycle Simulation Fixed Points

- No initial vector yields a loop.
- Out of $2^{11} = 2048$ potential steady states, only 7 are reached !

Basin si	ze	Cln3	MBF	SBF	Cln1,2	Cdh1	Swi5	Cdc20	Clb5,6	Sic1	Clb1,2	Mcm1
1,764		0	0	0	0	1	0	0	0	1	0	0
151		0	0	1	1	0	0	0	0	0	0	0
109		0	1	0	0	1	0	0	0	1	0	0
9		0	0	0	0	0	0	0	0	1	0	0
7		0	1	0	0	0	0	0	0	1	0	0
7		0	0	0	0	0	0	0	0	0	0	0
1		0	0	0	0	1	0	0	0	0	0	0

Table 1. The fixed points of the cell-cycle network

The main "attractor": this steady state attracts ~86% of initial states.

A Complete Cell-Cycle Simulation

• Start with a vector representing stationary G1 condition but with Cln3=1 (signal to initiate cell cycle).

Time	Cln3	MBF	SBF	Cln1,2	Cdh1	SWI5	Cdc20 and Cdc14	Clb5,6	SIc1	Clb1,2	Mcm1/SFF	Phase
1	1	0	0	0	1	0	0	0	1	0	0	START
2	0	1	1	0	1	0	0	0	1	0	0	G1
3	0	1	1	1	1	0	0	0	1	0	0	G1
4	0	1	1	1	0	0	0	0	0	0	0	G1
5	0	1	1	1	0	0	0	1	0	0	0	s
6	0	1	1	1	0	0	0	1	0	1	1	G ₂
7	0	0	0	1	0	0	1	1	0	1	1	м
8	0	0	0	0	0	1	1	0	0	1	1	м
9	0	0	0	0	0	1	1	0	1	1	1	м
10	0	0	0	0	0	1	1	0	1	0	1	м
11	0	0	0	0	1	1	1	0	1	0	0	м
12	0	0	0	0	1	1	0	0	1	0	0	G1
13	0	0	0	0	1	0	0	0	1	0	0	Stationary G ₁

The right column indicates the cell-cycle phases. Note that the number of time steps in each phase do not reflect its actual duration.

As indicated in the table, this simulation is compatible with the cell cycle stages: G₁ → S → G₂ → M → G_{1 (stationary)}

Cell-Cycle in Yeast - Transitions Tree

• Each node in the tree represents a vector, edges represent transitions in the simulation.



33

Transition tree for the main "attractor"

1764 = 86% of initial vectors.

(there are 6 other, smaller trees)

From <u>The yeast cell-cycle network is robustly designed</u>, Li et. al., PNAS 2004. Tree drawn with Pajek software (<u>http://vlado.fmf.uni-lj.si/pub/networks/pajek</u>)

Cell-Cycle in Yeast - Paper Conclusions

• The yeast cell-cycle is stable.

<u>Computational observation</u>: with high probability, changes to the initial vectors yield the same fixed point.

• The yeast cell-cycle is robust.

<u>Computational observation</u>: with high probability, small changes in the network structure (insert/delete node, change edge) will not harm cell cycle behavior.

Extensions to the Boolean Model

Extension 1: Discrete State Space

- Instead of 0/1, nodes can now assume states between 0,...,U
 (e.g. U=9)
- U=1 is the special case of the Boolean model we saw

• Transition function changes accordingly:

$$s_{i}(t+1) = \begin{cases} \min(U, s_{i}(t) + 1) & \text{if } \sigma_{i}(t) > 0\\ \max(0, s_{i}(t) - 1) & \text{if } \sigma_{i}(t) < 0\\ s_{i}(t) & \text{else} \end{cases}$$
$$\left(\sigma_{i}(t) = \sum_{i} w(j, i) \cdot s_{j}(t)\right)$$



Extension 2: State Update Function

- States change by ±1 no matter what.
- We may prefer the change to be proportional to $\sigma_i(t)$.

$$s_i(t+1) = \begin{cases} \min(U, s_i(t) + ?) & \text{if } \sigma_i(t) > 0\\ \max(0, s_i(t) - ?) & \text{if } \sigma_i(t) < 0\\ s_i(t) & \text{else} \end{cases}$$

• One reasonable option is a logarithmic order update:



Extensions 3, 4, ...

• A new type of interactions called dependency edges: nodes may block or enable other edges



If A>0 edge B \rightarrow D is blocked

- Delays on edges
 - The effect of node a on node b will occur in later steps

BioNSi – Biological Network Simulator

- Cytoscape is an open source software for visualization and analysis of networks and pathways (<u>www.cytoscape.org</u>).
- BioNSi is a plugin (app) of Cytoscape (<u>http://bionsi.wix.com/bionsi</u>)
 It extends the Boolean model in several ways, including those mentioned.



The circadian clock in mammalians in a day-night regime, in BioNSi

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BioNSi Hands-on Session

- Open Cytoscape (installed in this lab's computers)
- Go to bionsi.wix.com/bionsi
 - Click Download, save the file (BioNSi.jar) on your computer
- In Cytoscape's main menu:
 - click Apps \rightarrow App Manager \rightarrow Install from file
 - Choose BioNsi.jar file
 - BioNSi is now installed.
- In BioNSi's website, go to Examples \rightarrow Toy example
 - Open the pdf and follow the tutorial



Reflection: Constructing the "Right" Model?

• Main considerations in constructing mathematical models in biology:



Consideration	Meaning	Beware of
Modeling technique	Which model type is best suited for the data and goals of study?	Intractable approaches, too simplified, no sufficient data, etc.
Scope	What elements should be included in the model?	Excluding essential elements or including irrelevant ones
Detail	What level of detail should the model contain?	Too fine- or course-grained models
Parameters	How to set the model parameters appropriately?	Parameter overfitting

• Recall that even "incorrect" models may make correct predictions

Reflection: Iterative Simulation-Experiment Approach





Using models can point to gaps
in our biological understanding:
a model that fails to recapitulate
known biological data reveals
where our understanding needs
improvement.

Reflection: Discrete Models

- Computer Science is highly biased towards discrete notions, such as graphs (networks), strings (textual sequences), digital images, etc.
- Discrete notions and algorithms are highly underrepresented in life science curricula, where continuous notions (such as equations over the reals) and probability are taught more widely.





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